

# B

## Linearizing the Equation

Graphing is a tool for analyzing data obtained from a laboratory experiment. Almost all of your experimental work will involve one or more graphs. Therefore, it is important to learn how to properly construct and interpret graphs. The first step in analyzing your data is to decide how best to graph it.

### The Linear Graph

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The straight-line (linear) graph is the easiest type of graph to analyze because the functional relationship between the plotted variables is easily determined by comparing to the standard format of the equation

$$y = mx + b \tag{B.1}$$

where:

$y$  represents the dependent variable, plotted on the vertical axis

$x$  represents the independent variable, plotted on the horizontal axis

$m$  represents a constant quantity - whose value equals the slope

$b$  represents the intercept of the line with the y-axis when  $x = 0$

Additionally, a linear graph is the only graphical relationship that can be confirmed visually. It is usually possible to tell if a relationship is linear simply by looking at the plot - unlike curved graphs which could be power, exponential, or sinusoidal relationships. For example, if you were testing the relationship:

$$I = \frac{V}{R} \tag{B.2}$$

you could measure values for the current  $I$  for different values of the voltage  $V$ . Plotting  $I$  versus  $V$  should yield a straight line with slope equal to the inverse of resistance,  $\frac{1}{R}$ , and a y-intercept of 0.

## Linearization

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When two variables are plotted and the resulting graph is non-linear (power, exponential, or sinusoidal, for example), it is difficult to determine the functional relationship between the two variables from the shape of the curve. However, there are several techniques that can be used to turn a non-linear graph into a linear one, and they are discussed below.

### • Change of Variable Method

Suppose we are being asked to experimentally verify the following equation for the motion of a freely falling body:

$$v^2 = 2gh + v_o^2 \quad (\text{B.3})$$

Since the velocity  $v$  is a function of the height  $h$ , we collect data for  $v$  for varying values of  $h$ . Plotting the velocity directly against the height would give a *non-linear graph*, from which it is difficult to verify the above relationship.

However, if we were to plot a graph of  $v^2$  against  $h$ , we should obtain a linear graph with a slope equal to two times the acceleration due to gravity,  $2g$ , and a y-intercept equal to the initial velocity squared,  $v_o^2$ . A **change of variable** has been made so that  $v^2$  represents the y-axis. Since  $2g$  and  $v_o^2$  are constant values, we have:

$$v^2 = (\text{constant}) \times (h) + (\text{constant}) \quad (\text{B.4})$$

which agrees with the prescribed linear format:  $y = mx + b$ . This linear graph would then allow experimental verification of Equation B.3 and could be used to determine  $g$  and  $v_o$  for a falling body.

### • Log-Log Method for Power Relations

An equation of the form:

$$y = mx^A \quad (\text{B.5})$$

is called a power relation. This type of relation occurs frequently in physics and graphically yields a curve when  $y$  is plotted against  $x$ . However, it is very difficult to determine the exact value of the power  $A$  simply by looking at the shape of the curve. A simple technique, called the log-log method, solves this problem by showing whether a power relation actually holds, and if so, by giving the numerical value of  $A$ . As the name implies, this method involves taking log of both sides of the equation as follows:

$$\log y = A \log x + \log m \quad (\text{B.6})$$

By comparing Equation B.6 to  $y = mx + b$ , we see that a graph of  $\log y$  on the vertical axis versus  $\log x$  on the horizontal axis yields a straight line with slope  $A$ . Once  $A$  is known, we can then plot a graph of  $y$  versus  $x^A$  ("*Change of Variable*" method) which yields a straight line whose slope is the value of the constant  $m$ .

If the log-log graph is not linear, then the relation between  $y$  and  $x$  is obviously not a power relation, and we must try different methods to linearize the curve.

• **Semi-log Method for Exponential Relations**

A very common function is one where the change in the function (population increase, radioactive decay, etc.) is proportional to the function itself. This is an exponential function given by the general expression of the form:

$$y = Ae^{kx} \tag{B.7}$$

Taking the natural logarithm of both sides and rearranging gives:

$$\ln y = kx + \ln A \tag{B.8}$$

Comparing to  $y = mx + b$  we see that if we plot  $\ln y$  on the y-axis and  $x$  as the x-axis, the slope corresponds to  $k$  and the intercept corresponds to  $\ln A$ . Note that, unlike the log-log method, we plot  $\ln y$  versus  $x$ , instead of versus  $\ln x$ .

Once the correct variables for plotting a linear graph have been determined, the appropriate columns in the data table may be constructed.